



1. A ball is thrown vertically upward from the top of a 120 m tall building. It reaches its maximum height after 1.5 s. Ignore air resistance.

(a) Find the velocity of the ball just before it hits the ground. 3 points

(b) How long it takes for the ball to reach the ground? 1 point

**Solution:** We choose the **ground level** as **zero position**, and **upward** direction as the **positive** direction for all quantities.

First we need to calculate the initial velocity  $v_0$ . We have  $a = -9.8 \text{ m/s}^2$ , and at the maximum height  $v = 0$ . So

$$v = v_0 + at \implies v_0 = -at = 14.7 \text{ m/s}$$

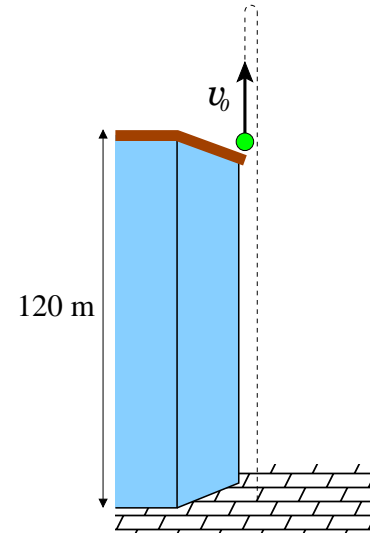
Hitting the ground: We have  $x_0 = 120 \text{ m}$ ,  $x = 0$ , and we use

$$v^2 = v_0^2 + 2a(x - x_0) \implies v^2 = 2568$$

$$\implies v = -50.7 \text{ m/s} \text{ [Negative, because it is falling down]}$$

Then

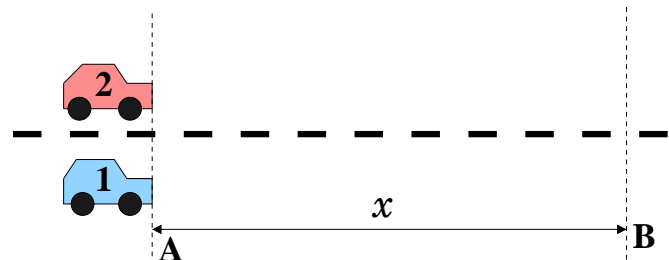
$$t = \frac{v - v_0}{a} = 6.7 \text{ s}$$



2. On a horizontal straight road, the car number 1 travelling with constant speed of  $v_1 = 30 \text{ m/s}$  passes car number 2 (at point **A**) at the instant ( $t=0$ ) car number 2 starts from rest with a constant acceleration  $a = 2.5 \text{ m/s}^2$ . After time  $t$  and travelling through a distance  $x$ , the two cars are together again (at point **B**).

(a) Find the time  $t$  that the cars needed to reach **B**. 3 points

(b) Find the distance  $x$  between the points **A** and **B**. 1 point



**Solution:** Both the cars have travelled the distance  $x$  in time  $t$ . For car **2** the initial speed is  $v_0 = 0$ . So

$$\left. \begin{array}{l} \text{For Car 1: } x = v_1 t \\ \text{For Car 2: } x = \frac{1}{2} a t^2 \end{array} \right\} \implies v_1 t = \frac{1}{2} a t^2 \implies t = \frac{2v_1}{a} = 24 \text{ s}$$

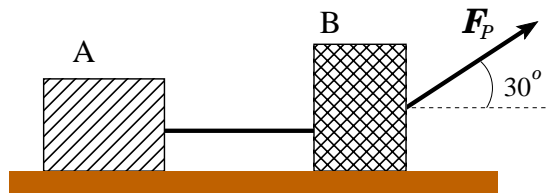
Then

$$x = v_1 t = 720 \text{ m}$$

3. The box A (mass  $M_A = 10$  kg) and the box B (mass  $M_B = 14$  kg) connected by a horizontal massless cord are on a frictionless horizontal surface. When a force  $F_P$  is applied to box B as shown, the boxes accelerate at  $a = 2.8$  m/s<sup>2</sup>.

- (a) Find the tension in the cord connecting the two boxes. 1 point

- (b) Find the magnitude of the applied force  $F_P$ . 2 points



**Solution:** Looking at the free-body diagram, and applying Newton's second law to the boxes, we have

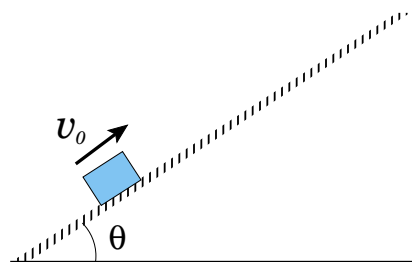
$$\text{Box A:} \quad F_T = M_A a \implies F_T = 28 \text{ N}$$

$$\text{Box B:} \quad F_P \cos 30^\circ - F_T = M_B a \implies F_P = \frac{M_B a + F_T}{\cos 30^\circ} = 78 \text{ N}$$

4. A block is given an initial speed of  $v_0 = 8$  m/s up the rough incline ( $\theta = 25^\circ$ ) as shown. The coefficient of kinetic friction between the incline and the block is  $\mu_k = 0.17$ . Find the time it takes the block to come momentarily to rest. 4 points

**Solution:** We have drawn the free-body diagram.

We choose the positive  $x$ -axis down the incline and the positive  $y$ -axis perpendicular to the incline.



Then, Newton's second law in the  $y$ -direction reads

$$F_N - Mg \cos \theta = 0 \implies F_N = Mg \cos \theta$$

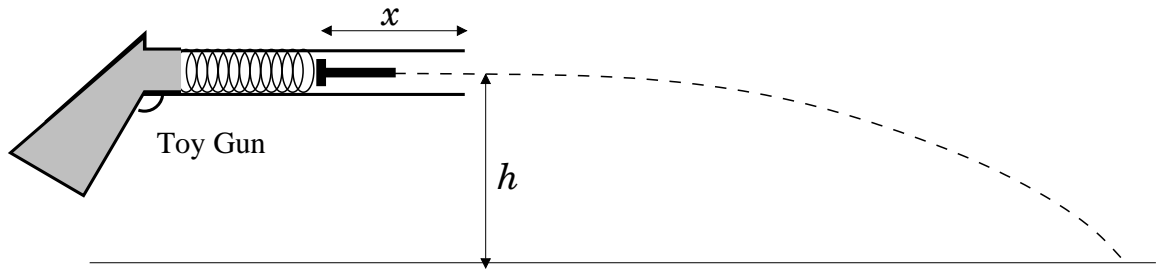
Newton's second law along the  $x$ -direction reads

$$\begin{aligned} Mg \sin \theta + F_{fr} &= Ma \implies Mg \sin \theta + \mu_k Mg \cos \theta = Ma \\ \implies a &= g(\sin \theta + \mu_k \cos \theta) = 5.65 \text{ m/s}^2 \end{aligned}$$

According to our sign convention,  $v_0 = -8$  m/s, and  $v = 0$ . So

$$v = v_0 + at \implies t = \frac{v - v_0}{a} = 1.4 \text{ s}$$

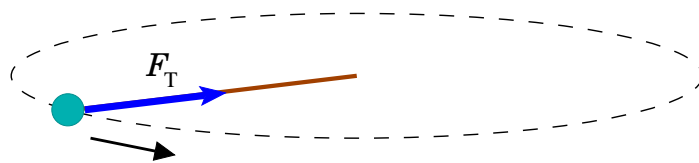
5. A toy gun is fitted with a spring of spring constant  $k = 200 \text{ N/m}$ . It is held horizontal at a height of  $h = 1.2 \text{ m}$  from the ground. The spring is compressed by  $x = 4 \text{ cm}$  by a dart of mass  $M = 0.017 \text{ kg}$ , and then it is released. Find the speed of the dart when it lands on the ground. Ignore air resistance. 3 points



**Solution:** The work-energy principle from start to finish is

$$\begin{aligned} \text{KE}_i + \text{PE}_i &= \text{KE}_f + \text{PE}_f \\ \implies 0 + \frac{1}{2}kx^2 + Mgh &= \frac{1}{2}Mv^2 + 0 + 0 \\ \implies 0.16 + 0.20 &= \frac{1}{2}Mv^2 \\ \implies v &= 6.5 \text{ m/s} \end{aligned}$$

6. A small ball of mass  $M = 0.15 \text{ kg}$  is connected to a cord. When the ball is in a horizontal uniform circular motion of radius  $R = 0.12 \text{ m}$ , the magnitude of the tension in the cord is  $F_T = 3 \text{ N}$ . Find the frequency of the ball's circular motion. 3 points



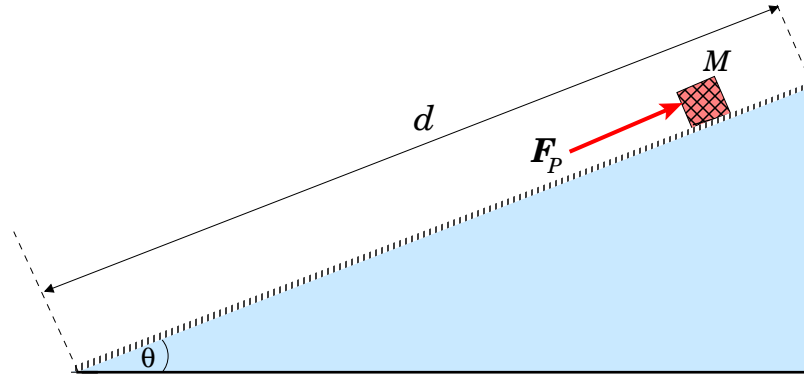
**Solution:** The Newton's law for the radial direction is

$$F_T = M \frac{v^2}{R} \implies v = \sqrt{\frac{F_T R}{M}} = 1.55 \text{ m/s}$$

Then

$$v = 2\pi R f \implies f = \frac{v}{2\pi R} = 2.06 \text{ Hz}$$

7. In a warehouse, boxes of mass  $M = 50$  kg slide down the rough incline ( $\theta = 15^\circ$ ) of length  $d = 6$  m. The coefficient of kinetic friction between the boxes and the incline is  $\mu_k = 0.12$ . The boxes start from rest at the top. For safety reasons, their maximum speed should be 2 m/s. A constant force of magnitude  $F_P$  applied parallel to the incline controls the speed of the boxes (see the figure). Find the minimum value of  $F_P$ . 5 points



**Solution:** The maximum height of the incline is

$$h = d \sin \theta = 1.55 \text{ m}$$

The work-energy principle from start to finish implies

$$KE_i + PE_i + W_{NC} = KE_f + PE_f$$

$$\Rightarrow 0 + Mgh - \mu_k Mg \cos \theta \times d - F_P d = \frac{1}{2} Mv^2 + 0$$

$$\Rightarrow 419 - F_P d = 100$$

$$\Rightarrow F_P = 53 \text{ N}$$