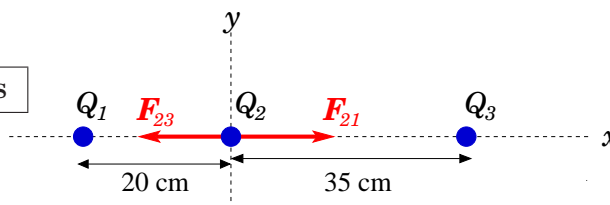




1. Three point charges,  $Q_1 = -4 \mu\text{C}$ ,  $Q_2 = -6 \mu\text{C}$  and  $Q_3 = -9 \mu\text{C}$  are on the  $x$ -axis as shown.

- (a) Find the magnitude of the net electric force on charge  $Q_2$ .

3 points



**Solution:** The forces  $\vec{F}_{21}$  and  $\vec{F}_{23}$  are shown in the figure (Note: Both the forces are repulsive). The  $x$ -components of the two forces are

$$F_{21,x} = +F_{21} = +\frac{k|Q_1Q_2|}{0.2^2} = +5.4 \text{ N}$$

$$F_{23,x} = -F_{23} = +\frac{k|Q_2Q_3|}{0.35^2} = -4.0 \text{ N}$$

So

$$F_{2,x} = F_{21,x} + F_{23,x} = +1.4 \text{ N}$$

- (b) The direction of the net force on  $Q_2$  is: (Tick the correct answer)

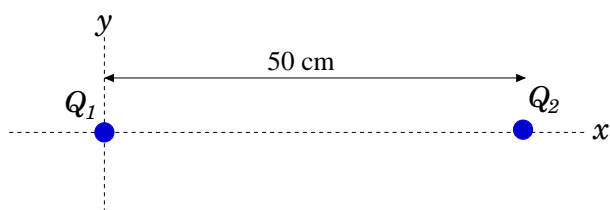
1 point

In the **Positive**  $x$ -direction ☒

In the **Negative**  $x$ -direction ☐

2. Two point charges,  $Q_1 = +3 \mu\text{C}$  and  $Q_2 = -9 \mu\text{C}$  are on the  $x$ -axis as shown. Find the  $x$ -coordinate of the point  $P$  where the net electric field  $\vec{E} = 0$ .

4 points



**Solution:** Since,  $Q_1$  and  $Q_2$  have opposite signs, and  $|Q_1| < |Q_2|$ , the point  $P$  at which the net electric field  $\vec{E} = 0$ , must be to the **left** of  $Q_1$ , *i.e.*  $x < 0$ . Let  $d$  = distance of  $P$  from the origin (from  $Q_1$ ). Then

$$E_1 = E_2 \implies \frac{k|Q_1|}{d^2} = \frac{k|Q_2|}{(0.5 + d)^2} \implies \frac{0.5 + d}{d} = \sqrt{\frac{|Q_2|}{|Q_1|}} = 1.73$$

We chose the positive square root because on the left-hand-side all the terms are positive. Solving for  $d$  we get  $d = 0.68 \text{ m}$ . Then the coordinate of  $P$  is  $x = -0.68 \text{ m}$ .

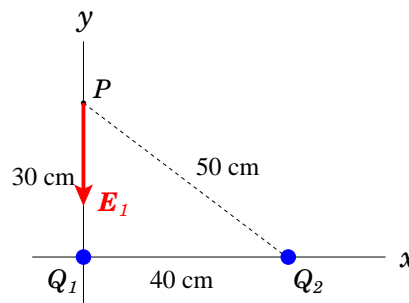
3. Two point charges,  $Q_1 = -6 \text{ nC}$  and an unknown  $Q_2$  are placed in the  $xy$ -plane as shown. The  $y$ -component of the net electric field at the point  $P$  is  $E_y = -276 \text{ N/C}$ .

(a) Find the charge  $Q_2$  (with correct sign).

**2+1 points**

(b) Find the  $x$ -component of the net electric field at the point  $P$

**2 points**

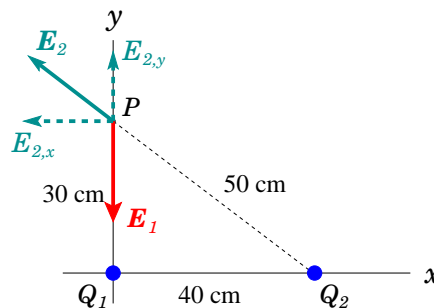


**Solution:** We know  $Q_1$  (it is negative), so we can draw  $\mathbf{E}_1$  at  $P$ .

The  $y$ -component of the net electric field is

$$E_y = E_{1,y} + E_{2,y} \implies -276 = -\frac{k|Q_1|}{0.3^2} + E_{2,y} \implies E_{2,y} = +324 \text{ N/C}$$

Since  $E_{2,y}$  is positive, the only way  $\mathbf{E}_2$  can be is as shown in the figure here. This makes  $Q_2 > 0$ .



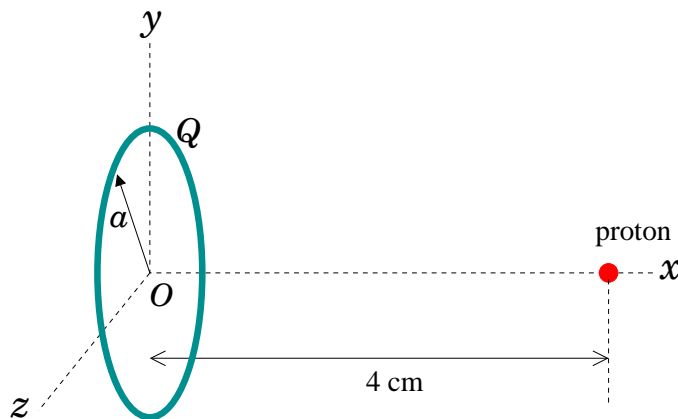
To get  $Q_2$ ,

$$E_{2,y} = +\frac{k|Q_2|}{0.5^2} \times \frac{0.3}{0.5} = +324 \implies |Q_2| = 1.5 \times 10^{-8} \text{ C} \implies Q_2 = +1.5 \times 10^{-8} \text{ C}$$

Now

$$E_x = E_{2,x} = -\frac{k|Q_2|}{0.5^2} \times \frac{0.4}{0.5} \implies E_x = -432 \text{ N/C}$$

4. A uniformly charged ring of radius  $a = 1.5$  cm is fixed in the  $yz$ -plane with its centre at the origin  $O$ . A proton is **released from rest** on the  $x$ -axis at  $x = 4$  cm (see the figure). The proton **passes through**  $O$  with speed  $v = 3.3 \times 10^6$  m/s. Find the charge  $Q$  on the ring. 6 points



**Solution:** The work-energy principle is

$$KE_i + PE_i = KE_f + PE_f$$

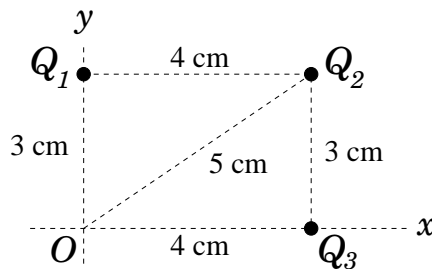
$$0 + e \times V_{Ring}(x = 4\text{cm}) = \frac{1}{2}m_p v^2 + e \times V_{Ring}(x = 0)$$

$$\Rightarrow e \times \frac{kQ}{(0.04^2 + 0.015^2)^{1/2}} = \frac{1}{2}m_p v^2 + e \times \frac{kQ}{0.015}$$

$$\Rightarrow (3.37 \times 10^{-8}) Q = 9.09 \times 10^{-15} + (9.6 \times 10^{-8})$$

$$\Rightarrow -(6.23 \times 10^{-8}) Q = (9.6 \times 10^{-8}) \Rightarrow \boxed{Q = -1.46 \times 10^{-7} \text{ C}}$$

5. Three point charges,  $Q_1$ ,  $Q_2$  and  $Q_3$  are in the  $xy$ -plane as shown. If  $Q_1 = +6.0$  nC,  $Q_2 = +5.0$  nC and the **electric potential**  $V = 0$  at the origin  $O$ , find the **electric potential energy** ( $PE_3$ ) of the charge  $Q_3$ . 3 points



**Solution:** At the origin  $O$ ,  $V = 0$ ,

$$\frac{kQ_1}{0.03} + \frac{kQ_2}{0.05} + \frac{kQ_3}{0.04} = 0 \Rightarrow Q_3 = -12.0 \text{ nC}$$

The electric potential energy of  $Q_3$  is

$$PE_3 = \frac{kQ_1Q_3}{0.05} + \frac{kQ_2Q_3}{0.03} = -3.1 \times 10^{-5} \text{ J}$$

6. A parallel-plate capacitor of plate-area  $A = 4.0 \times 10^{-4} \text{ m}^2$  and thickness  $d = 1.4 \text{ mm}$  is filled with paraffin (dielectric constant,  $K = 3.4$ ). It is connected to a battery of voltage  $V = 6 \text{ V}$ . How much **electric energy** is stored in the capacitor? 3 points

**Solution:** The capacitance is

$$C = K\epsilon_0 \frac{A}{d} = 8.6 \times 10^{-12} \text{ F}$$

$$\text{PE} = \frac{1}{2} CV^2 = 1.55 \times 10^{-10} \text{ J}$$

7. A cellulose-filled parallel-plate capacitor has  $\text{PE} = 150 \text{ nJ}$  of energy when it is connected to a battery. While it is still **connected to the battery**, the **cellulose slab is pulled out** and the **thickness is reduced** from  $d = 4 \text{ mm}$  to  $d' = 2 \text{ mm}$ . The dielectric constant of cellulose is  $K=1.5$ . How much energy is now stored in the capacitor? 4 points

**Solution:** Let the old capacitor be  $C$  and the new capacitor be  $C'$ . Then

$$\frac{C'}{C} = \left(\frac{1}{K}\right) \left(\frac{A}{A}\right) \left(\frac{4 \times 10^{-3}}{2 \times 10^{-3}}\right) \Rightarrow \frac{C'}{C} = 1.333$$

In this case, the voltage  $V$  across the capacitor remains unchanged. So

$$\left. \begin{array}{l} \text{PE} = \frac{1}{2} CV^2 \\ \text{PE}' = \frac{1}{2} C'V^2 \end{array} \right\} \Rightarrow \frac{\text{PE}'}{\text{PE}} = \frac{C'}{C} = 1.33 \Rightarrow \text{PE}' = 1.333 \times \text{PE} = 200 \text{ nJ}$$

8. Three capacitors are connected to a battery of voltage  $\varepsilon = 20$  V as shown. The capacitance  $C_1 = 16$  nF. The energies stored in  $C_1$  and  $C_2$  are respectively  $PE_1 = 200$  nJ and  $PE_2 = 360$  nJ.

(a) Find the energy  $PE_3$  stored in the capacitor  $C_3$ .

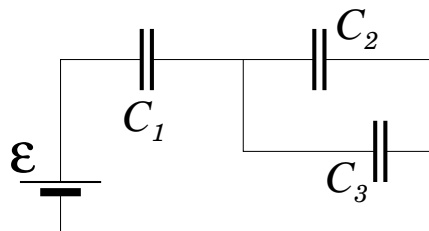
**3 points**

(b) Find the capacitance  $C_2$ .

**3 points**

(c) Find the capacitance  $C_3$ .

**1 point**



**Solution:** We have

$$PE_1 = \frac{Q_1^2}{2C_1} \implies Q_1 = \sqrt{2C_1 \times PE_1} = 8.0 \times 10^{-8} \text{ C}$$

$$\left. \begin{array}{l} C_2 \text{ and } C_3 \text{ are parallel} \\ C_1 \text{ and } C_{23} \text{ are in series} \end{array} \right\} \implies Q_{123} = Q_1 = 8.0 \times 10^{-8} \text{ C}$$

Total energy in the circuit is:  $PE = \frac{1}{2} Q_{123} \varepsilon = 8.0 \times 10^{-7} \text{ J}$

So, energy in  $C_3$  is:  $PE_3 = PE - PE_1 - PE_2 = 2.4 \times 10^{-7} \text{ J}$

Voltage across  $C_1$  is:  $V_1 = \frac{Q_1}{C_1} = 5 \text{ V}$

$$\left. \begin{array}{l} C_2 \text{ and } C_3 \text{ are parallel} \\ C_1 \text{ and } C_{23} \text{ are in series} \end{array} \right\} \implies V_2 = V_3 = V_{23} = \varepsilon - V_1 = 15 \text{ V}$$

Then  $PE_2 = \frac{1}{2} C_2 V_2^2 \implies C_2 = \frac{2 \times PE_2}{V_2^2} = 3.2 \times 10^{-9} \text{ F}$

And  $PE_3 = \frac{1}{2} C_3 V_3^2 \implies C_3 = \frac{2 \times PE_3}{V_3^2} = 2.1 \times 10^{-9} \text{ F}$