



Final Examination
Spring Semester 2024 – 2025

May 19, 2025
Time: 2:00 – 4:00 PM

Name: Student No: Sec. No: Serial No:

Instructors: Drs. Almumin, Lajko, Sharma, and Vagenas

Fundamental constants

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N.m}^2/\text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$	(Permeability of free space)
$ e = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

For use by Instructors only

Problems	1	2	3	4	5	6	7	8	9	10	Questions	Total
Marks												

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. Cheating incidents will be processed according to the university rules.

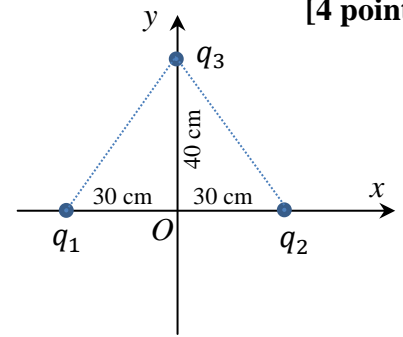
PART I: Solve the following problems. Show your solutions in detail.

1. Three point charges, $q_1 = -6 \mu\text{C}$, and $q_3 = q_2 = 6 \mu\text{C}$, are placed on the xy -plane, as shown. Calculate the x and y components of the net electric force, \vec{F}_3 , acting on q_3 . [4 points]

$$r_{13} = r_{23} = \sqrt{(0.3 \text{ m})^2 + (0.4 \text{ m})^2} = 0.5 \text{ m}$$

$$F_{3,y} = F_{13,y} + F_{23,y} = -k \frac{|q_1 q_3|}{r_{13}^2} \sin(\theta) + k \frac{|q_2 q_3|}{r_{23}^2} \sin(\theta) = 0$$

$$F_{3,x} = F_{13,x} + F_{23,x} = -2k \frac{|q_1 q_3|}{r_{13}^2} \cos(\theta) = -2k \frac{|q_1 q_3|}{r_{13}^2} \frac{3}{5} = -1.6 \text{ N}$$



2. A spherical surface has uniform surface charge density σ and radius $a = 10 \text{ cm}$. A point charge $q = 9 \text{ nC}$ is placed at its center. At $b = 8 \text{ cm}$ from the center, at point B , the magnitude and direction of electric field \vec{E}_B is the same as the electric field \vec{E}_C at point C , that is at distance $2b$ from the center. Find the value of the surface charge density σ . [4 points]

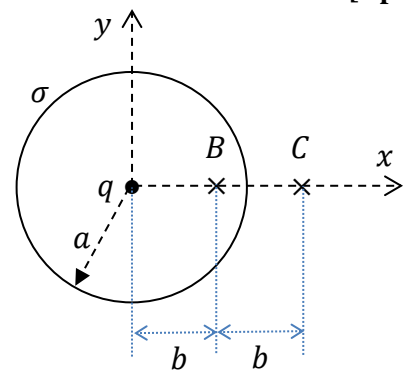
Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0}$$

$$E(4\pi b^2) = Q_{enc}^B / \epsilon_0 = q / \epsilon_0$$

$$E(4\pi (2b)^2) = Q_{enc}^C / \epsilon_0 = 4q / \epsilon_0 = (q + \sigma 4\pi a^2) / \epsilon_0$$

$$\text{So } \sigma = \frac{3q}{4\pi a^2} = 215 \text{ nC/m}^2$$

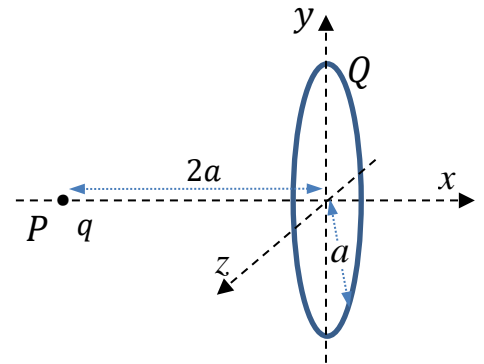


3. A ring of radius $a = 0.8 \text{ m}$ has charge $Q = 12 \text{ } \mu\text{C}$ uniformly distributed on it and it is placed in the yz -plane, as shown. A point charge $q = -8 \text{ } \mu\text{C}$, $m = 10^{-3} \text{ kg}$, is released from rest at point P . Calculate the speed of the point charge at the center of the ring. [3 points]

$$E_{in} = E_{fin} \Rightarrow qV_P = qV_C + m \frac{v^2}{2}$$

$$V_P = \frac{kQ}{\sqrt{(2a)^2 + a^2}} = \frac{kQ}{\sqrt{5}a}; \quad V_C = \frac{kQ}{a}$$

$$\frac{2}{m} q (V_P - V_C) = v^2 \Rightarrow v = 34.6 \text{ m/s}$$



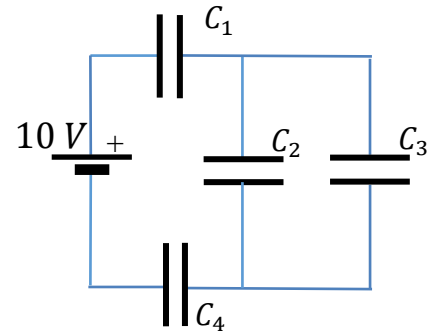
4. A network of identical capacitors with $C_1 = C_2 = C_3 = C_4 = 6 \text{ } \mu\text{F}$ is charged by a battery, as shown. Determine the electric potential across C_3 . [4 points]

$$C_{23} = C_2 + C_3 = 12 \text{ } \mu\text{F}$$

$$C_{1234} = \left(\frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4} \right)^{-1} = 2.4 \text{ } \mu\text{F}$$

$$Q = C_{1234} V = 24 \text{ } \mu\text{C}$$

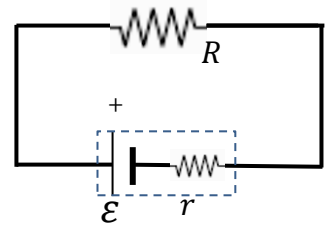
$$V_3 = \frac{Q}{C_{23}} = 2 \text{ V}$$



5. In the circuit below, the power dissipation in the internal resistor is 4.5 W. If $\mathcal{E} = 30 \text{ V}$ and $r = 2 \Omega$, calculate the value of R . [3 points]

$$P_r = I^2 r = 4.5 \text{ W} \Rightarrow I = 1.5 \text{ A}$$

$$I = \frac{\mathcal{E}}{r+R} \Rightarrow R = 18 \Omega$$

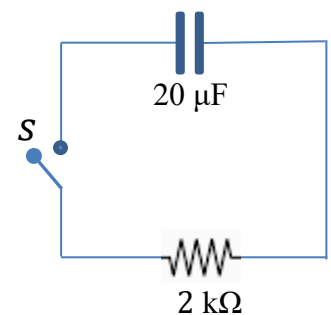


6. In the RC circuit below, the initial charge on the capacitor is $Q_0 = 60 \mu\text{C}$. At time $t = 0 \text{ s}$, the switch S is closed. Find the time at which the energy stored in the capacitor is $\frac{1}{9}$ of its initial value. [3 points]

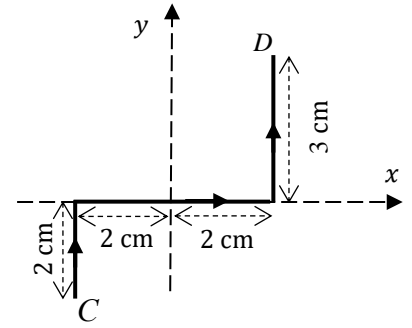
$$q(t) = Q_0 e^{-\frac{t}{RC}} \Rightarrow U(t) = \frac{q^2(t)}{2C} = \frac{Q_0^2 e^{-\frac{2t}{RC}}}{2C} = U_0 e^{-\frac{2t}{RC}}$$

$$\frac{U_0}{9} = U_0 e^{-\frac{2t}{RC}} \Rightarrow \frac{1}{9} = e^{-\frac{2t}{RC}} \Rightarrow$$

$$\ln\left(\frac{1}{9}\right) = -\frac{2t}{RC} \Rightarrow t = -\frac{RC}{2} \ln\left(\frac{1}{9}\right) = 43.9 \text{ ms}$$



7. A wire carries a current $I = 8.0 \text{ A}$ from point C to point D , as shown. Calculate the magnetic force \vec{F} acting on the wire due to a uniform magnetic field $\vec{B} = (2.0 \text{ T})\hat{k}$. [3 points]

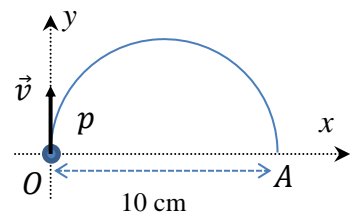


$$\vec{L}_{eff} = (\Delta x)\hat{i} + (\Delta y)\hat{j} = (0.04 \text{ m})\hat{i} + (0.02 \text{ m} + 0.03 \text{ m})\hat{j}$$

$$\vec{F} = I\vec{L}_{eff} \times \vec{B} = 8.0 \text{ A}[(0.04 \text{ m})\hat{i} + (0.05 \text{ m})\hat{j}] \times [(2.0 \text{ T})\hat{k}]$$

$$\vec{F} = (0.8 \text{ N})\hat{i} - (0.64 \text{ N})\hat{j}$$

8. A proton at point O moves momentarily with a velocity $\vec{v} = (2 \times 10^6 \frac{\text{m}}{\text{s}})\hat{j}$ in a uniform magnetic field and follows a semi-circular path from O to A , as shown. Find the magnitude and direction of the magnetic field that makes the proton to move on the semi-circular path. [3 points]

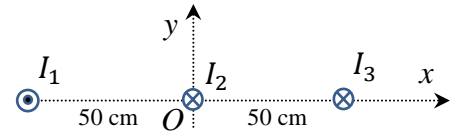


$$R = \frac{0.1 \text{ m}}{2} = 0.05 \text{ m}$$

$$R = \frac{mv_{\perp}}{|q|B} \Rightarrow B = \frac{mv_{\perp}}{|q|R} = 0.42 \text{ T}$$

Direction: out of page or \hat{k} .

9. Three long parallel wires carry currents $I_1 = 8 \text{ A}$, $I_2 = 2 \text{ A}$, $I_3 = 2 \text{ A}$ in the directions as shown. Calculate the force per unit length acting on wire 3. **[3 points]**

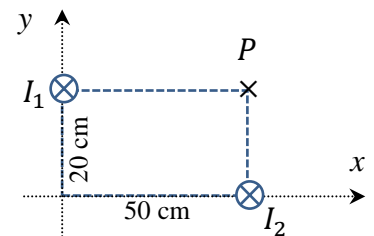


$$\frac{\vec{F}_{13}}{L} = \frac{\mu_0 I_1 I_3}{2\pi 1m} \hat{i} = 3.2 \times 10^{-6} \frac{\text{N}}{\text{m}} \hat{i}$$

$$\frac{\vec{F}_{23}}{L} = -\frac{\mu_0 I_2 I_3}{2\pi 0.5m} \hat{i} = -1.6 \times 10^{-6} \frac{\text{N}}{\text{m}} \hat{i}$$

$$\frac{\vec{F}_3}{L} = \frac{\vec{F}_{13}}{L} + \frac{\vec{F}_{23}}{L} = 1.6 \times 10^{-6} \frac{\text{N}}{\text{m}} \hat{i}$$

10. Two very long wires are perpendicular to the xy -plane and carry currents $I_1 = 10 \text{ A}$, and $I_2 = 8 \text{ A}$, as shown. Calculate the magnitude and direction of the net magnetic field \vec{B} at point P . **[4 points]**



$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi 0.5m} \hat{j} = -(4.0 \times 10^{-6} \text{T}) \hat{j}$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi 0.2m} \hat{i} = (8.0 \times 10^{-6} \text{T}) \hat{i} \Rightarrow$$


$$\vec{B}_{net} = (8.0 \times 10^{-6} \text{T}) \hat{i} - (4.0 \times 10^{-6} \text{T}) \hat{j}$$

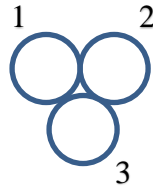
$$B = \sqrt{B_1^2 + B_2^2} = 8.9 \times 10^{-6} \text{T}$$

$$\theta = \tan^{-1} \left[\frac{B_{net,y}}{B_{net,x}} \right] = -26.6^\circ$$


PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

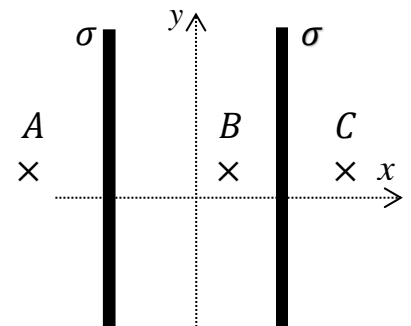
1. Three *identical* conducting spheres are initially far from each other and have charges $Q_1 = 70\text{ C}$, $Q_2 = 30\text{ C}$, and $Q_3 = -40\text{ C}$, respectively. Then, all three spheres are touched to each other, as shown. The final charge on sphere 3 will be

- a) 0 C.
b) 10 C.
c) 20 C. 
d) 40 C.




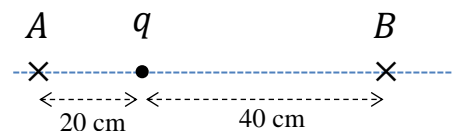
2. Two very large parallel sheets are placed perpendicular to the xy -plane and have uniform surface charge densities σ . Which statement is correct for the net electric field?

- a) The net electric field is zero at point A.
b) The net electric field is zero at point B. 
c) The net electric field is zero at point C.
d) The net electric field is zero at point A, B, and C.




3. Points A and B are close to a negative point charge, $q < 0$, as shown. Take $V = 0$ at infinity. Which relation is correct for the electric potential at points A and B?

- a) $V_A = V_B$.
b) $V_A < V_B$. 
c) $V_A > V_B$.
d) $V_A = -2V_B$.



4. Two air-filled parallel-plate capacitors C_1 and C_2 have identical separation but C_2 has twice the plate area as C_1 . Both are charged to the same charge Q . If the energy stored in the capacitors C_1 and C_2 are U_1 and U_2 , respectively, which relation is correct?

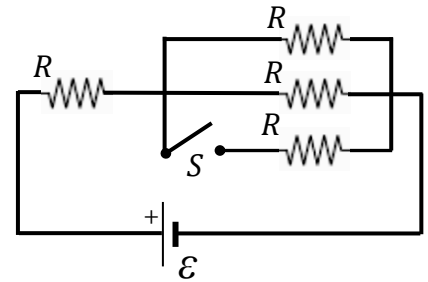
- a) $U_1 = U_2$.
b) $2U_1 = U_2$.
c) $U_1 = 4U_2$.
d) $U_1 = 2U_2$. 

5. A cylindrical copper wire A has length L , radius r , and resistance R_A . Another cylindrical copper wire B has length $2L$, radius $2r$, and resistance R_B . Which relation is correct?

- a) $R_A = 2R_B$. ←
- b) $R_A = R_B$.
- c) $R_A = 4R_B$.
- d) $2R_A = R_B$.

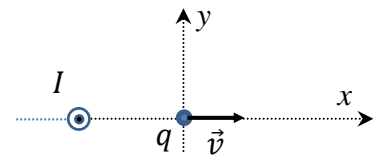
6. In the circuit below, the switch S is initially closed and then it is opened. The current from the emf device will

- a) remain the same.
- b) decrease. ←
- c) increase.
- d) increase if $R > 10\ \Omega$, and will decrease if $R < 10\ \Omega$.



7. A very long wire carries a current I , and a positive point charge q moves as shown. Which statement is correct for the direction of the force acting on the point charge?

- a) It is along $+\hat{j}$.
- b) It is along $-\hat{j}$.
- c) It is along $+\hat{k}$. ←
- d) It is along $-\hat{k}$.



8. The magnetic field in the solenoid is

- a) inversely proportional to the number of turns.
- b) inversely proportional to the turn per unit length.
- c) independent of the turn per unit length.
- d) proportional to the turn per unit length. ←