



Second Midterm Examination Spring Semester 2024 - 2025

April 24, 2025

Time: 6:30 PM – 8:00 PM

Name: Student No:

Section No: Serial No:

Instructors: Drs. Al-Munin, Lajko, Sharma, & Vagenas

Fundamental constants

$k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$	(Coulomb constant)
$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7} \text{ T.m/A}$	(Permeability of free space)
$ e = 1.60 \times 10^{-19} \text{ C}$	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8 \text{ m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31} \text{ kg}$	(Electron mass)
$m_p = 1.67 \times 10^{-27} \text{ kg}$	(Proton mass)

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^3$	$M = 10^6$	$G = 10^9$	$T = 10^{12}$

For use by Instructors only

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

Instructions to the Students:

1. Mobile or other electronic devices are **strictly prohibited** during the exam.
2. Programmable calculators, which can store equations, are not allowed.
3. Cheating incidents will be processed according to the university rules.

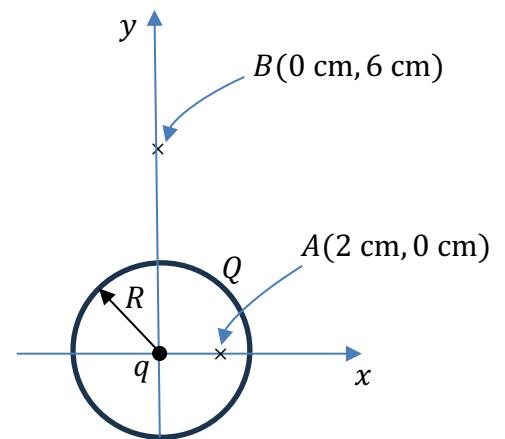
PART I: Solve the following problems. Show your solutions in detail.

1. A conducting thin spherical shell of radius $R = 3$ cm is centered at the origin of the xy -axis and has electric charge $Q = +30$ nC. A point charge $q = -5$ nC is placed at the center of the spherical shell, as shown. Find the potential difference $V_A - V_B$. **[5 points]**

$$V_A = V_q + V_Q \Rightarrow V_A = k \frac{q}{0.02} + k \frac{Q}{R} \Rightarrow V_A = +6750 \text{ V}$$

$$V_B = V_q + V_Q \Rightarrow V_B = k \frac{q}{0.06} + k \frac{Q}{0.06} \Rightarrow V_B = +3750 \text{ V}$$

$$V_A - V_B = +3000 \text{ V}$$



2. An air-filled parallel-plate capacitor has plate area $A = 5$ cm² and separation $d = 2$ mm. After charging, the energy density between the plates of the capacitor is 450 μJ/m³. Find the charge Q stored in the capacitor. **[3 points]**

$$u = \frac{U}{(\text{volume})} \Rightarrow U = u (Ad) \Rightarrow U = 4.5 \times 10^{-10} \text{ J}$$

$$C = \epsilon_0 \frac{A}{d} \Rightarrow C = 2.2125 \times 10^{-12} \text{ F}$$

$$U = \frac{1}{2} \frac{Q^2}{C} \Rightarrow Q = \sqrt{2UC} \Rightarrow Q = 44.6 \text{ pC}$$

3. An air-filled parallel-plate capacitor with plate area A and separation d , has capacitance $C_0 = 10 \text{ nF}$. The capacitor is partially filled with two dielectric materials of dielectric constant $K_1 = 4$ and $K_2 = 5$, as shown. Each dielectric material fills half of the space between the capacitor plates. Find the capacitance now. **[4 points]**

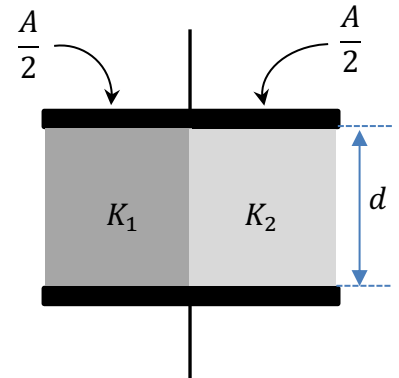
$$C_0 = \epsilon_0 \frac{A}{d}$$

$$C_1 = K_1 \epsilon_0 \frac{A/2}{d} = \frac{K_1}{2} C_0 = 2C_0 = 20 \text{ nF}$$

$$C_2 = K_2 \epsilon_0 \frac{A/2}{d} = \frac{K_2}{2} C_0 = 2.5C_0 = 25 \text{ nF}$$

C_1 and C_2 are in parallel

$$C = C_1 + C_2 \Rightarrow C = 45 \text{ nF}$$



4. A cylindrical wire of gold with length $L = 5 \text{ m}$ is connected to a battery of 10 V . The resistivity of gold is $2.44 \times 10^{-8} \Omega \cdot \text{m}$ and the concentration of free electrons for gold is $8.85 \times 10^{28} \text{ m}^{-3}$. Calculate the drift speed v_d of the electrons. **[3 points]**

$$E = \frac{V}{L} \Rightarrow E = 2 \text{ N/C}$$

$$E = \rho J \Rightarrow J = \frac{E}{\rho} \Rightarrow J = 8.197 \times 10^7 \text{ A/m}^2$$

$$J = n|q|v_d \Rightarrow v_d = \frac{J}{n|e|} \Rightarrow v_d = 57.9 \times 10^{-4} \text{ m/s}$$

5. A wire carries a time-dependent electric current $I(t) = I_0 e^{-5t}$ (electric current is in ampere and time is in second). If the total charge that flows through the wire in the time interval $0 \leq t \leq 0.4$ s is $Q=0.7$ C, find the value of I_0 . [3 points]

$$I(t) = \frac{dQ}{dt} \Rightarrow dQ = I(t)dt \Rightarrow \int_0^Q dQ = \int_0^{0.4s} I(t)dt$$

$$Q = \int_0^{0.4s} I_0 e^{-5t} dt \Rightarrow Q = I_0 \int_0^{0.4s} e^{-5t} dt$$

$$Q = I_0 \frac{e^{-5t}}{(-5)} \Big|_0^{0.4s} \Rightarrow Q = \frac{I_0}{(-5)} (e^{-2} - 1) \Rightarrow I_0 = 4.05 \text{ A}$$

6. Find the electric currents in all branches in the circuit below. [5 Points]

Junction rule:

$$I_1 = I_2 + I_3 \quad (1)$$

Upper loop

$$10 - 6I_2 - 9I_1 = 0 \Rightarrow 10 - 9I_1 - 6I_2 = 0 \quad (2)$$

Lower loop:

$$20 - 8I_3 + 6I_2 = 0 \Rightarrow 20 - 8(I_1 - I_2) + 6I_2 = 0 \Rightarrow$$

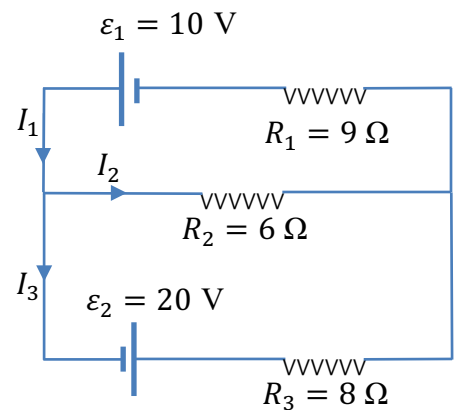
$$20 - 8I_1 + 14I_2 = 0 \Rightarrow 10 - 4I_1 + 7I_2 = 0 \quad (3)$$

Multiply (2) with (+7): $70 - 63I_1 - 42I_2 = 0$

Multiply (3) with (+6): $60 - 24I_1 + 42I_2 = 0$

Add the two above equations: $130 - 87I_1 = 0 \Rightarrow I_1 = 1.494 \text{ A}$

(3) $\Rightarrow I_2 = -0.574 \text{ A}$ and (1) $\Rightarrow I_3 = 2.068 \text{ A}$



7. In the circuit below, calculate the power dissipated on the 7-Ω resistor.

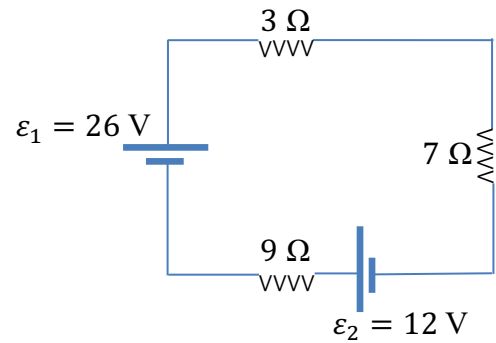
[3 Points]

Loop rule:

$$26 - 3I - 7I + 12 - 9I = 0$$

$$38 - 19I = 0 \Rightarrow I = 2 \text{ A}$$

$$P_{7\Omega} = I^2 \times 7 \Rightarrow P_{7\Omega} = 28 \text{ W}$$



8. In the circuit below, the switch S is closed at time $t = 0$ s. At time t_1 , the electric current is the one third of the maximum current of the electric circuit. Find the electric charge at time t_1 . [4 Points]

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = 2 \Omega$$

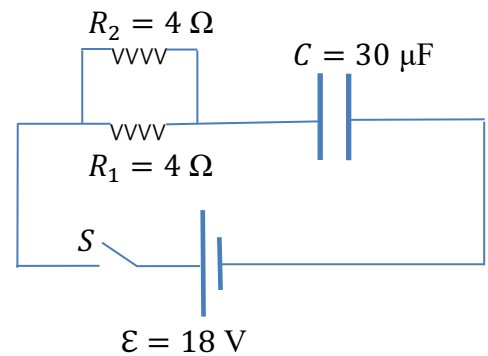
$$Q_f = C\varepsilon \Rightarrow Q_f = 540 \mu\text{C}$$

$$q(t) = Q_f[1 - e^{-\frac{t}{R_{eq}C}}]$$

$$i(t) = \frac{dq(t)}{dt} = \frac{Q_f}{R_{eq}C} e^{-\frac{t}{R_{eq}C}}$$

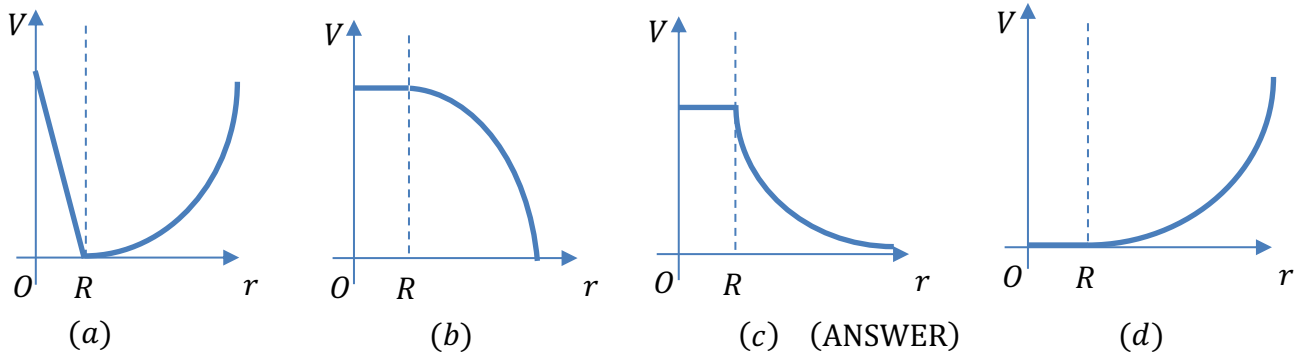
$$i(t_1) = \frac{Q_f}{R_{eq}C} e^{-\frac{t_1}{R_{eq}C}} \Rightarrow \frac{I_0}{3} = I_0 e^{-\frac{t_1}{RC}} \Rightarrow e^{-\frac{t_1}{RC}} = \frac{1}{3}$$

$$q(t_1) = Q_f[1 - e^{-\frac{t_1}{R_{eq}C}}] \Rightarrow q(t_1) = 540[1 - \frac{1}{3}] \mu\text{C} \Rightarrow q(t_1) = 360 \mu\text{C}$$



PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

1. The graph of the potential as a function of the distance from the center of a positively charged spherical conductor with radius R is:

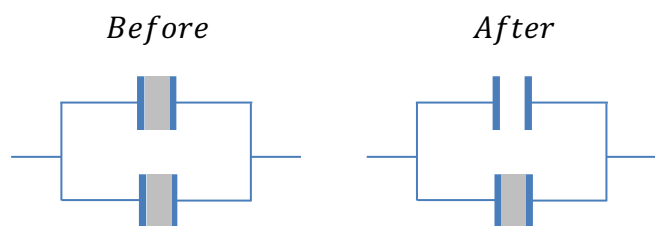


2. An air-filled parallel-plate capacitor with plate separation d , plate area A , and capacitance C is charged by a battery V and the battery is then disconnected. If the plate area A is decreased, the electric energy will

- a) be zero.
b) increase. (ANSWER)
c) decrease.
d) remain the same.

3. Two identical air-filled parallel-plate capacitors connected in parallel are fully filled with dielectric slabs of dielectric constant K . If the dielectric slab is removed from one of them, as shown, the capacitance of the network will now

- a) be undetermined.
b) increase.
c) decrease. (ANSWER)
d) remain the same.



4. The current density in metals is

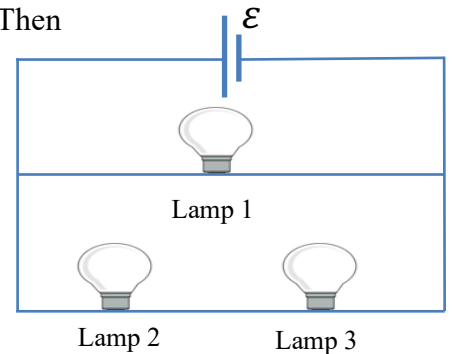
- a) a complicated function of the applied electric field.
b) not related to the applied electric field.
c) inversely proportional to the applied electric field.
d) proportional to the applied electric field. (ANSWER)

5. The junction rule is a consequence of the

- a) conservation of energy.
- b) conservation of electric potential.
- c) conservation of electric charge. (ANSWER)
- d) conservation of electric potential energy.

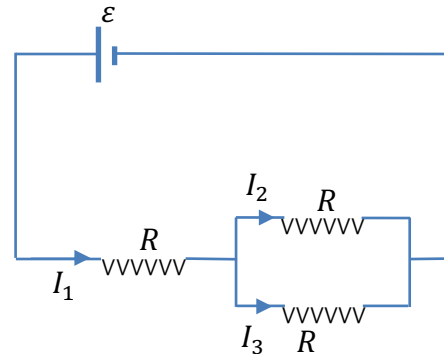
6. In the below circuit, if a new lamp is connected in series with Lamp 1. Then

- a) lamps 1, 2, and 3 will light more.
- b) lamps 1, 2, and 3 will light less.
- c) lamp 1 will light less but 2 and 3 will light more.
- d) lamp 1 will light less, but 2 and 3 will light the same. (ANSWER)



7. In the circuit below, all resistances are the same. Which is the correct relation for the electric currents?

- a) $I_1 = I_2 = I_3$
- b) $I_1 = I_2 = 2I_3$
- c) $I_1 = \frac{I_2}{2} = \frac{I_3}{2}$
- d) $\frac{I_1}{2} = I_2 = I_3$ (ANSWER)



8. In an RC circuit of time constant $\tau = RC$, the capacitor is completely empty and at time $t = 0$, its charging starts. At time $t = \tau$, the electric current I

- a) decreases to 36.8% of its initial value. (ANSWER)
- b) decreases to 63.2% of its initial value.
- c) increases to 36.8% of its initial value.
- d) increases to 63.2% of its initial value.